1990 – AB1

1. A particle, initially at rest, moves along the x-axis so that its acceleration at any time $t \geq 0$ is given by $a(t) = 12t^2 - 4$. The position of the particle when $t = 1$ is $x(1) = 3$.

(a) Find the values of t for which the particle is at rest.

(b) Write an expression for the position $x(t)$ of the particle at any time $t \geq 0$.

(c) Find the total distance traveled by the particle from $t = 0$ to $t = 2$.

1990 – AB2

2. Let f be the function given by $f(x) = \ln\left(\frac{x}{x-1}\right)$.

(a) What is the domain of f?

(b) Find the value of the derivative of f at $x = -1$.

(c) Write an expression for $f^{-1}(x)$, where f^{-1} denotes the inverse function of f.

1990 – AB3

3. Let R be the region enclosed by the graphs of $y = e^x$, $y = (x-1)^2$, and the line $x = 1$.

(a) Find the area of R.

(b) Find the volume of the solid generated when R is revolved about the <u>x-axis</u>.

(c) Set up, but <u>do not integrate</u>, an integral expression in terms of a single variable for the volume of the solid generated when R is revolved about the <u>y-axis</u>.

1990 - AB4

4. The radius r of a sphere is increasing at a constant rate of 0.04 centimeters per second.
(Note: The volume of a sphere with radius r is $V = \frac{4}{3}\pi r^3$.)

(a) At the time when the radius of the sphere is 10 centimeters, what is the rate of increase of its volume?

(b) At the time when the volume of the sphere is 36π cubic centimeters, what is the rate of increase of the area of a cross section through the center of the sphere?

(c) At the time when the volume and the radius of the sphere are increasing at the same numerical rate, what is the radius?

1990 - AB5

5. Let f be the function defined by $f(x) = \sin^2 x - \sin x$ for $0 \le x \le \frac{3\pi}{2}$.

(a) Find the x-intercepts of the graph of f.

(b) Find the intervals on which f is increasing.

(c) Find the absolute maximum value and the absolute minimum value of f. Justify your answer.

1990 - AB6

6. Let f be the function that is given by $f(x) = \dfrac{ax+b}{x^2-c}$ and that has the following properties.

(i) The graph of f is symmetric with respect to the y-axis.

(ii) $\lim\limits_{x \to 2^+} f(x) = +\infty$

(iii) $f'(1) = -2$

(a) Determine the values of a, b, and c.

(b) Write an equation for each vertical and each horizontal asymptote of the graph of f.

(c) Sketch the graph of f in the xy-plane provided below.

1990 – BC 1

1. A particle starts at time $t = 0$ and moves along the x-axis so that its position at any time $t \geq 0$ is given by $x(t) = (t - 1)^3(2t - 3)$.

   (a) Find the velocity of the particle at any time $t \geq 0$.

   (b) For what values of t is the velocity of the particle less than zero?

   (c) Find the value of t when the particle is moving and the acceleration is zero.

1990 – BC 2

2. Let R be the region in the xy-plane between the graphs of $y = e^x$ and $y = e^{-x}$ from $x = 0$ to $x = 2$.

   (a) Find the volume of the solid generated when R is revolved about the x-axis.

   (b) Find the volume of the solid generated when R is revolved about the y-axis.

1990 – BC 3

3. Let $f(x) = 12 - x^2$ for $x \geq 0$ and $f(x) \geq 0$.

   (a) The line tangent to the graph of f at the point $(k, f(k))$ intercepts the x-axis at $x = 4$. What is the value of k?

   (b) An isosceles triangle whose base is the interval from (0, 0) to $(c, 0)$ has its vertex on the graph of f. For what value of c does the triangle have maximum area? Justify your answer.

1990 - BC 4

Let R be the region inside the graph of the polar curve $r = 2$ and outside the graph of the polar curve $r = 2(1 - \sin\theta)$.

(a) Sketch the two polar curves in the xy-plane provided below and shade the region R.

(b) Find the area of R.

1990 - BC 5

5. Let f be the function defined by $f(x) = \frac{1}{x - 1}$.

(a) Write the first four terms and the general term of the Taylor series expansion of $f(x)$ about $x = 2$.

(b) Use the result from part (a) to find the first four terms and the general term of the series expansio about $x = 2$ for $\ln|x - 1|$.

(c) Use the series in part (b) to compute a number that differs from $\ln\frac{3}{2}$ by less than 0.05. Justify your answer.

1990 - BC 6

6. Let f and g be continuous functions with the following properties.

(i) $g(x) = A - f(x)$ where A is a constant

(ii) $\int_1^2 f(x)dx = \int_2^3 g(x)dx$

(iii) $\int_2^3 f(x)dx = -3A$

(a) Find $\int_1^3 f(x)dx$ in terms of A.

(b) Find the average value of $g(x)$ in terms of A, over the interval $[1, 3]$.

(c) Find the value of k if $\int_0^1 f(x + 1)dx = kA$.